# CIS7 Unit 9 In-Class Assignment

Refer to Chapter 12 and 13 in textbook or lecture notes for lesson concepts and information.

1. Define **Mathematic Induction Principle.**

Mathematic Induction is a technique used to prove specific statements about natural numbers directly. It can be applied to a wide variety of circumstances. It’s a way of proving mathematical statements by saying if the first case is true then all other cases must also be true.

1. Use Induction Principle to prove that 2n < n!, for n ≥ 4.

Base Case: n = 4

24 = 16 < 4! = 24; True

Induction Step:

Let n = k

Let k ≥ 4

Assume P(K) is true, that is 2k < k! (induction hypothesis)

Prove P(K+1) is also true, that is 2(k+1) < (k+1)!

2k < k! [by induction hypothesis]

24 < 4! (True)

(2 \* 2 \* 2 \* 2) < (4 \* 3 \* 2 \* 1)

2 \* 2k < 2 \* k! [multiply both sides by 2, as 2(k+1) is (2\*2k)

2 \* 2k < 2 \* 4! (True)

32 < 48

2 \* k! < (k+1)! [(k+1) is larger than 2]

2 \* 4! < (4+1)!

48 < 5 \* 4 \* 3 \* 2 \* 1 (True)

(k+1) \* k! = (k+1)! [by definition of factorial]

(4+1) \* 4! = (4+1)!

5 \* 4 \* 3 \* 2 \* 1 = 5! (True)

Thus, we have proven that our claim is true.

1. Use Induction Principle to prove is (3n−1) a multiple of 2 for n ≥ 1.

Base Case: N = 1

(31 – 1) = 2

(32 – 1) = 8

Induction Step:

Let n = k

Let k ≥ 1

Assume that P(K) is true, that is (3k - 1) results in an even number or number that is multiple of 2 [induction hypothesis]

Prove P(K+1) is true, that is (3(k+1) – 1) results in an even number or number that is multiple of 2

3k – 1 [by induction hypothesis]

31 -1 = 2 (True)

(3(k+1) – 1) = 3 \* 3k – 1

3(1+1) – 1 = 3 \* 31 – 1 (True)

8 = 8

(3(k+1) – 1) = 2 \* 3k + (3k -1) [multiple of two, continue from previous term]

3(1+1) – 1 = 2 \* 31 + 31 -1 (True)

8 = 8

Thus, we have proven that our claim is true.

1. Define Recurrence.

A recurrence relation is an equation that expresses each element of a sequence as a function of the preceding ones.

1. Find a recurrence relation and initial conditions for: 1, 5,17,53,161,485…

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Term | 1 | 5 | 17 | 53 | 161 | 485 |
| Difference |  | 4 | 12 | 36 | 108 | 324 |
| Common Factor |  |  | 4 \* 3 | 12 \* 3 | 36 \* 3 | 108 \* 3 |

We notice that it appears to be growing by a factor of 3. We check if this is the original sequence. 1 \* 3 = 3, 5 \* 3 = 15, 17 \* 3 = 51 and so on. It appears that we always end up 2 less than the next term.

an = 3an-1 + 2

= 3 \* 1 + 2 = 5

1. Use Recurrence Relation concept to check that an=2n+1 is a solution to the recurrence relation an=2an−1−1 with a1=3.

an = 2n + 1

a1 = 21 + 1 = 3

a2 = 22 + 1 = 5

a3 = 23 + 1 = 9

a4 = 24 + 1 = 17

an = 2an-1 – 1

a1 = 3

a2 = 2 \* 3 – 1 = 5

a3 = 2 \* 5 – 1 = 9

…

an = 2n + 1 is a solution to the recurrence relation a­n = 2an-1 – 1 with a1 = 3

1. Solve the recurrence relation an=a n−1+n with initial term a0=4.

an = an-1 + n

a0 = 4

a1 = 4 + 1 = 5

a2 = 5 + 2 = 7

a3 = 7 + 3 = 10

…

Difference between terms:

a1 – a0 = 5 – 4 = 1

a2 – a1 = 7 – 5 = 2

a3 – a2 = 10 – 7 = 3

…

an – an-1 = n

Find sum of both sides

(a1 – a0) + (a2 – a1) + (a3 – a2) + … + (an – an-1) = 1 + 2 + 3 + 4 + … + n

an – an-1 = n [Rearrange]

an = an-1 + n

an = (n(n+1))/2 + a0

an = (n(n+1))/2 + 4

Test Conditions:

a0 = 4

a1 = (1(1+1))/2 + 4 = 5

**Solution of recurrence relation, subject to the initial condition is: an = (n(n+1))/2 + 4**